Early Journal Content on JSTOR, Free to Anyone in the World

This article is one of nearly 500,000 scholarly works digitized and made freely available to everyone in the world by JSTOR.

Known as the Early Journal Content, this set of works include research articles, news, letters, and other writings published in more than 200 of the oldest leading academic journals. The works date from the mid-seventeenth to the early twentieth centuries.

We encourage people to read and share the Early Journal Content openly and to tell others that this resource exists. People may post this content online or redistribute in any way for non-commercial purposes.

Read more about Early Journal Content at http://about.jstor.org/participate-jstor/individuals/early-journal-content.

JSTOR is a digital library of academic journals, books, and primary source objects. JSTOR helps people discover, use, and build upon a wide range of content through a powerful research and teaching platform, and preserves this content for future generations. JSTOR is part of ITHAKA, a not-for-profit organization that also includes Ithaka S+R and Portico. For more information about JSTOR, please contact support@jstor.org.

Denote by the symbol $\{x, p\}$ a pair of numbers such that

(*i*) x and p are positive integers;

(*ii*) $x<p$;

(*iii*) $1+xp$ is a factor of $(p^2-1)^2$.

From one such pair we may derive a right-neighboring pair $\{p, y\}$, where $y=pk-x$. Since $x<p$, $k>1$, we have $p<y$, so that properties (*i*) and (*ii*) hold for $\{p, y\}$. Property (*iii*) is true by (3_2). Hence any pair leads to a chain of right-neighboring pairs:

$$\{x, p\},\ \{p, pk-x\},\ \{pk-x, (pk-x)k-p\}\ldots(4).$$

For $x=1$, $k=p-1$, by (1). Hence we have the successive pairs

$$\{1, k+1\},\ \{k+1, k^2+k+1\},\ \{k^2+k-1, k^3+k^2-2k-1\}\ldots(5).$$

When the second member of a pair (5) is a prime p, the first member is a solution x $(x<p)$. We proceed to show that every pair $\{X, P\}$ leading to a solution and having P prime may be obtained from the pairs (5) by assigning a suitable value to k. To this end we consider the pair $\{w, x\}$, $w=xk-p$, which is left-neighboring to a given pair $\{x, p\}$. We determine the conditions under which $\{w, x\}$ has the properties (*i*)—(*iii*). The third property holds in view of (3_1). By the latter,

$$wx=A-1,\ A\equiv\frac{(x^2-1)^2}{1+px},\ w=\frac{x^3-2x-p}{1+px}<\frac{x^3}{x^2},$$

since $x<p$. Hence $w<x$, and property (*ii*) holds. Finally, (*i*) holds, viz., w is positive, unless $A=0$ or 1. Hence we may form in succession left-neighboring pairs until we reach a pair with $A=0$ or 1. One of the latter cases must ultimately present itself, since a series of decreasing positive integers must terminate. If $A=0$, then $x=1$, and the chain contains the first pair (5), so that $\{X, P\}$ occurs in the list (5). If $A=1$, then $p=xk$, $k=x^2-2$. The first terms of the pairs (4) are then

$$x,\ x(x^2-2),\ x(x^2-2)^2-x,\ x(x^2-2)^3-2x(x^2-2),\ \ldots$$

a series of increasing integers with the factor x, so that no one is a prime P.

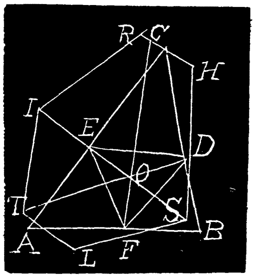
MECHANICS.

210. Proposed by W. J. GREENSTREET, M. A., Marling School, Stroud, England.

A rigid triangle is formed of three weightless, smoothly jointed, rigid rods BC, CA, AB. At their mid points D, E, F, respectively, are small, smooth rings, through which passes an endless, stretched, elastic string, forming the triangle DEF. Find by graphical construction the reaction at the joints.

Solution by G. B. M. ZERR, A. M., Ph. D., 4243 Girard Avenue, Philadelphia, Pa.

Let ABC be the triangle, DEF the stretched string, P the force exerted by the string, O the in-center of DEF. Then the resultant of the forces in DE, DF is $2P\cos\frac{1}{2}A$, acting from D through O. Let OT represent the this force. The resultant of the forces in FE, FD is $2P\cos\frac{1}{2}C$ acting from F through O. Let OR represent this force. The resultant of the forces in EF, ED is $2P\cos\frac{1}{2}B$ acting from E through O. Let OS represent this force. OT can be replaced by two parallel forces equal to $\frac{1}{2}OT$ acting at B and C, respectively; OR, by two parallel forces equal to $\frac{1}{2}OR$ acting at A and B, respectively; and OS, by two parallel forces equal to $\frac{1}{2}OS$ acting at A and C, respectively. Completing the parallelograms, we get OH the resultant of OR and OS, and $\frac{1}{2}OH$ represents the reaction at A. OI is the resultant of OR and OT while $\frac{1}{2}OI$ represents the reaction at B. OL is the resultant of OT and OS, and $\frac{1}{2}OL$ represents the reaction at C.



AVERAGE AND PROBABILITY.

---

194. Proposed by PROF. R. D. CARMICHAEL, Anniston, Ala.

What is the mean value of the triangle formed by joining three points taken at random on the circumference of a circle?

Solution by G. B. M. ZERR, A. M., Ph. D., 4243 Girard Avenue, Philadelphia, Pa.

Let OC=diameter of given circle$=2a$. Let the point P be fixed, draw PO perpendicular to OC, and draw OA and OB. Let $\angle POA=\theta>\frac{1}{2}\pi$, $\angle POB=\phi<\frac{1}{2}\pi$.

$\therefore OA=2a\sin\theta$, and $OB=2a\sin\phi$. Area $OAB=2a^2\sin\theta\sin\phi\sin(\theta-\phi)$. Average area$=\triangle$,

$$=\frac{\int_0^\pi\int_0^\theta 2a^2\sin\theta\sin\phi\sin(\theta-\phi)d\theta\,d\phi}{\int_0^\pi\int_0^\theta d\theta\,d\phi}$$

$$=\frac{4a^2}{\pi^2}\int_0^\pi\int_0^\theta \sin\theta\sin\phi\sin(\theta-\phi)d\theta\,d\phi=\frac{2a^2}{\pi^2}\int_0^\pi(\sin^2\theta-\theta\sin\theta\cos\theta)d\theta$$

$$=\frac{3a^2}{2\pi}.$$